Modeling MHD Nanofluid Flow over a Moving Surface Influenced by Thermal Radiation Using HAM

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ABSTRACT

In this paper the effect of Brownian motion and Thermophoresis on boundary layer flow of MHD nanofluid flow over a moving surface with the influence of thermal radiation are studied The governing partial differential equations are transformed into ordinary differential equations, by using a similarity transformation. The resulting ordinary differential equations are solved by homotopy analysis method. We observed physical insight and interesting aspects of the problem in presence of thermal radiation. The effect of different non-dimensional parameters are studied namely Prandtle number Pr, Brownian motion parameter N_b Thermophoresis parameter $N_{t,}$ Lewis number Le, Hartman number Ha and Radiation parameter R are depicted graphically for velocity, temperature and nanoparticle concentration profile.

Keywords: *Thermophorosis, Brownian motion, Nanofluid, Thermal radiation, HAM, MHD.*

I. INTRODUCTION

Heat can be transferred from one place to another by conduction in solids, convection in liquids or gases and radiation for which no media is required. Thermal radiation is the fundamental mechanisms of heat transfer. Heat generation and heat transfer are very important characteristic phenomenon involved in many engineering and industrial process. Olanerwaju et al [16] gave the concept of boundary layer flow of nanofluids over a moving surface in presence of thermal radiation. In some industries enormous heat will be generated and transferred through thermal radiation which may lead to many damages in process of mechanics. To reduce this thermal radiation one has to use some coolant in machineries.

Nanofluids are made of nanoparticles (<100nm) suspended in a base fluids such as water, oil and ethylene glycol. Recently nanotechnology has invented nanofluids discovered by Choi's team [4]. There exist a nano layer that acts as a thermal bridge between solid nanoparticles and base fluid which plays significant role in heat transfer. It can be used as coolant in industries resulting in energy saving. Ahmadreza [22] observed that due to their higher thermal conductivity nanofluids are used as liquid cooling. Azwadi Che Sidik [27] noted the applications of nanofluids and various minichannel configurations for heat transfer improvement. Nanofluids have heat transfer applications as a microelectronic fuel cells, electronic cooling, domestic refrigerator chillers, solar water heating chillers, heat pipes, lubrication, oil recovery, detergency and processes of soil remediation,. Husam Abdulrasool Hasan et al. [28] discussed the heat transfer enhancement using nanofluids for Cooling.

One of the recent Bio-medical applications used to kill cancer cells is Nano-cryosurgery which is also called cryotherapy or cryodestruction or cryoabalation. Jing Liu and Zhong-Shan Deng [12] highlighted the advances and challenges in Nano-cryosurgery. De-Rui Di et al. [18] explained experimentally that cryosurgery uses freezing to destroy undesirable tissues by using MgO nanoparticles were adopted as a biodegradable nontoxic agent. Chandran and Senthil Kumar [20] explained that in surgical procedure by using cryosurgery device called cryoprobe in cancer treatment to destroy undesirable tumor cells. By introduction of nanoparticles into target tissue area, it improves image contrast and gives the guidance for Cryosurgery.

Crane [2] investigated the concept of fluid flow over a stretching sheet. The flow of various non-Newtonian fluids over stretching sheet was analyzed by Liao [9]. Concept of boundary layer flow over a moving surface was presented by Sakiadis [1].

In many physical and engineering applications such as plasma physics, packed-bed catalytic reactors, thermal insulation, geothermal reservoirs etc, the .applications of MHD boundary layer flow of heat and mass transfer are found. Bachok et al. [13] studied boundary layer flow of nanofluid over a moving surface. Anuar Ishak [16] studied the effect of radiation on MHD boundary layer flow of a viscous fluid over an exponentially stretching sheet. Shadloo et al. [15] applied HAM to obtain the analytical solutions in the study of a two-dimensional steady convective flow of a micro and engineering fluid flows such as polymeric fluids, liquid crystals, paints colloidal fluids. Eshetu Haile and Shankar [23, 26] studied boundary layer flow of nanofluids through a porous medium subjected to a magnetic field, thermal radiation, viscous dissipation and chemical reaction effects. They observed that Brownian effect and thermophorosis effect improves the heat transfer rate. Nick Kaiser [8] explained that the Stefan-Boltzmann law gives the energy of thermal radiation emitted per unit time by a black body of a surface area which is directly proportional to fourth power of the temperature.

II. BASIC EQUATIONS

Consider the steady MHD boundary layer flow of a nanofluid in a uniform free stream in presence of thermal radiation whose velocity is assumed as U, towards a moving surface velocity is given by Uw = λU where λ is the velocity parameter. λ > 0 corresponds to the downstream movement of the plate from the origin and λ < 0 corresponds to the upstream movement of the plate.The flow being confined at $y \geq 0$. it is assumed that y co-ordinate is measured normal to the moving surface and uniform magnetic field is applied in 'y' direction. Let Tw and Cw are the temperature of fluid and nanoparticle fraction at wall. For free stream values are taken to be T_∞ and C_∞. Tiwari and Das [11] proposed the nanofluid model with Boussinesq and boundary layer approximation. The geometry of flow is shown in figure1. Continuity, momentum and energy equation for the steady MHD boundary layer flow of a nanofluid past a moving semi infinite flat plate in a uniform stream in presence of thermal radiation are given by

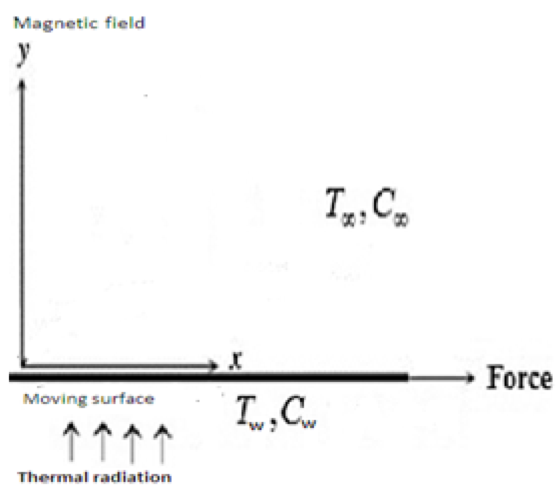

Figure1: Physical model and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_f} u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{\alpha}{k}\frac{\partial q_r}{\partial y} + \tau\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where u is the velocity component along the x-axis, v is the velocity component along the y-axis, ν is the kinematic viscosity coefficient, k is the thermal conductivity, q_r radiative heat flux, B0 is the uniform magnetic field strength of the base fluid, σ is the electrical conductivity of the base fluid, τ is the ratio of the nanoparticle heat capacity and the base fluid heat capacity, q_w is the heat flux , q_m Mass flux, α = k/$(\rho c)_f$ is the thermal diffusivity of the fluid. D_B the Brownian diffusion coefficient, D_T the Thermophoresis diffusion coefficient, The associated boundary conditions are taken as

$$v = 0, u = \lambda U, T = T_W, C = C_W \ at\ y = 0 \quad (5)$$

$$U \to U,\ T \to T_\infty, C \to C_\infty \ as\ y \to \infty. \quad (6)$$

By using Rosseland diffusion approximation [5, 6, 8] the radiative heat flux qr is given by

$$q_r = \frac{-4\sigma^*}{3K_s}\frac{\partial T^4}{\partial y}, \quad (7)$$

Where $\sigma^*$and K_s are the Stefan-Boltzman constant and the Rosseland mean absorption coefficient respectively. We assume that the temperature differences within the flow are sufficiently small such that T4 may be expressed as a linear function of temperature

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \quad (8)$$

Using (7) and (8) in the last term of equation (3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3K_s}\frac{\partial^2 T}{\partial y^2}. \quad (9)$$

To reduce the governing equations into a system of ordinary differential equations, we introduce the following Similarity transformations.

$$\Psi = (2U\nu x)^{\frac{1}{2}}\phi(\eta), \quad \psi(\eta) = \frac{T-T_\infty}{T_W-T_\infty}, \quad (10)$$

$$\xi(\eta) = \frac{C-C_\infty}{C_W-C_\infty}, \qquad \eta = (U/2\nu x)^{1/2} y. \quad (11)$$

where $\phi(\eta)$, $\psi(\eta)$ and $\xi(\eta)$ are the dimensionless stream function, temperature, and nanoparticle concentration respectively and η is the similarity variable.
The stream function Ψ is defined as

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x}. \quad (12)$$

Using similarity transformation and associated boundary conditions, the continuity equation is identically satisfied. Momentum equation and energy equation reduces to ODE.
The governing coupled non linear equations for this problem are

$$\phi''' + \phi\phi'' - \mathrm{Ha}(\phi') = 0, \quad (13)$$

$$\frac{(3+4R)}{Pr}\psi'' + \phi\psi' + N_b\,\xi'\psi' + N_t\psi'^2 = 0, \quad (14)$$

$$\xi'' + \mathrm{Le}\,\phi\xi' + \frac{N_t}{N_b}\psi'' = 0, \quad (15)$$

$$\phi(0) = 0, \phi'(0) = \lambda, \phi' \to 1 \text{ as } \eta \to \infty, \quad (16)$$

$$\psi(0) = 1, \psi \to 0 \text{ as } \eta \to \infty, \quad (17)$$

$$\xi(0) = 1, \xi \to 0 \ as\ \eta \to \infty, \quad (18)$$

where

$$R = \frac{4\alpha\delta T_\infty^3}{kk},\ \Pr = \frac{\nu}{\alpha},\ \mathrm{Le} = \frac{\nu}{D_B}, \quad (19)$$

$$Ha = \frac{2xB_0^2}{U\rho_f}, N_b = \frac{(\rho c)_p D_B(\varphi_W - \varphi_\infty)}{\nu(\rho c)_f}, \quad N_t = \frac{(\rho c)_p D_T(T_W - T_\infty)}{\nu(\rho c)_f T_\infty} \quad (20)$$

Nb is the Brownian motion parameter, Nt is the Thermophoresis parameter, τ_w is the Shear stress, R the Radiation parameter, Pr the Prandtl number, Le Lewis number, Ha the Hartman number

Limiting case: Stanford Shately and Jagadish Prakash [24] in which spectral relaxation method is applied directly to ODE. Eshetu Haile and Shankar B [23, 26] in which shooting technique along with the fourth order Runge-Kutta integration scheme method is applied directly to non linear ODE. But here we applied semi analytical method HAM to ODE directly, which exactly matches with earlier result

III. METHODOLOGY

Homotopy Analysis Method for nonlinear boundary value problems are discussed in [10, 21].

The region of convergence and the rate of convergence can be adjusted by varying auxiliary linear operator L, auxiliary parameter h and initial solution. Liao [10] determined the optimal value of h involves plotting the h-curves of the solution. We can except to see the horizontal over the range for which the solution converges. In figure 2 we have plotted $\phi''(0)$ against auxiliary parameter h. similarly we can obtain for temperature profile and nanoparticle profile.

The governing coupled non linear equations for this problem are written as

$$N[\phi(\eta)] = \phi''' + \phi\phi'' - \mathrm{Ha}(\phi'), \quad (21)$$

$$N[\psi(\eta)] = \frac{(3+4R)}{Pr}\psi'' + \phi\psi' + N_b\,\xi'\psi' + N_t\psi'^2, \quad (22)$$

$$N[\xi(\eta)] = \xi'' + \mathrm{Le}\,\phi\xi' + \frac{N_t}{N_b}\psi''. \quad (23)$$

For HAM solution an auxiliary linear operator for the equation (21), (22), (23) respectively as

$$L_\phi = \frac{\partial^3}{\partial\eta^3} + \frac{\partial^2}{\partial\eta^2},\ L_\psi = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta},\ L_\xi = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta}. \quad (24)$$

Consider $L_\phi[\phi] = 0$, $L_\psi[\psi] = 0$, $L_\xi[\xi] = 0$ and using boundary conditions (16), (17), (18) for $\boldsymbol{\phi}$, $\boldsymbol{\psi}$, ξ we get the initial approximations are $\phi_0(\eta), \Psi_0(\eta), \xi_0(\eta)$ as

$$\phi_0(\eta) = f_0(\eta) = (\lambda - 1) + \eta + (1 - \lambda)e^{-\eta}, \quad (25)$$

$$\Psi_0(\eta) = \theta_0(\eta) = e^{-\eta}, \quad (26)$$

$$\xi_0(\eta) = \varphi_0(\eta) = e^{-\eta}. \quad (27)$$

As p varies from 0 to 1, the solution $\phi_0(\eta), \Psi_0(\eta), \xi_0(\eta)$ varies from the initial guess to the exact solution $\phi(\eta)$, $\psi(\eta)$, $\xi(\eta)$.

Homotopy equations for (27), (28), (29) are constructed as below

$$(1-p)L[A(\eta,p) - \phi_0(\eta)] = hp\left\{\frac{\partial^3 A}{\partial\eta^3} + A\frac{\partial^2 A}{\partial\eta^2} - \mathrm{Ha}\left(\frac{\partial A}{\partial\eta}\right)\right\}, \quad (28)$$

$$(1-p)L[B(\eta,p) - \Psi_0(\eta)] = hp\left\{\left(\frac{3+4R}{Pr}\right)\frac{\partial^2 B}{\partial\eta^2} + A\frac{\partial B}{\partial\eta} + N_b\frac{\partial C}{\partial\eta}\frac{\partial B}{\partial\eta} + N_t\left(\frac{\partial B}{\partial\eta}\right)^2\right\}, \quad (29)$$

$$(1-p)L[D(\eta,p) - \xi_0(\eta)] = hp\left\{\frac{\partial^2 D}{\partial\eta^2} + \mathrm{Le}A\frac{\partial D}{\partial\eta} + \frac{N_t}{N_b}\frac{\partial^2 B}{\partial\eta^2}\right\}. \quad (30)$$

When p=0 and p=1 we have

$$A(\eta,0) = \phi_0(\eta) \qquad A(\eta,1) = \phi(\eta), \quad (31)$$

$$B(\eta,0) = \psi_0(\eta) \qquad B(\eta,1) = \psi(\eta), \quad (32)$$

$$D(\eta,0) = \xi_0(\eta) \qquad D(\eta,1) = \xi(\eta). \quad (33)$$

Obviously conditions are,

$$A(0,p) = 0, A_\eta(0,p) = \lambda,\ A_\eta(\infty,p) = 1, \quad (34)$$

$$B(0,p) = 1, B(\infty,p) = 0, \quad (35)$$

$$D(0,p) = 1, D(\infty,p) = 0. \quad (36)$$

Applying Maclaurin's series expansion to $A(\eta,p)$, $B(\eta,p)$ and $D(\eta,p)$ and using initial approximation (25)-(27) we get

$A(\eta, p) = f_0(\eta) + \sum_{k=1}^{\infty} f_k(\eta) p^k,$ (37)
$B(\eta, p) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_k(\eta) p^k,$ (38)
$D(\eta, p) = \varphi_0(\eta) + \sum_{k=1}^{\infty} \varphi_k(\eta) p^k.$ (39)

The convergence region of the above series depends upon the auxiliary linear operator L, and the non-zero auxiliary parameter h which are to be selected such that solution converges at p = 1.
$\phi(\eta) = f_0(\eta) + \sum_{k=1}^{\infty} f_m(\eta),$ (40)
$\psi(\eta) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_m(\eta),$ (41)
$\xi(\eta) = \varphi_0(\eta) + \sum_{k=1}^{\infty} \varphi_m(\eta).$ (42)

Differentiating equation (28), (29) and (30) m times about the embedding parameter p, using Leibnitz theorem, setting p = 0 and dividing by m! We get
$L[f_m - \chi_m f_{m-1}] = hR_m(\eta),$ (43)
$L[\theta_m - \chi_m \theta_{m-1}] = hS_m(\eta),$ (44)
$L[\varphi_m - \chi_m \varphi_{m-1}] = hT_m(\eta).$ (45)

where $\chi_m = \begin{cases} 0 & \text{when } m \leq 1 \\ 1 & \text{when } m > 1 \end{cases}$ and
$R_m(\eta) = f'''_{m-1}(\eta) + \sum_{k=0}^{m-1} f_{m-1-k}(\eta)\, f''_k(\eta) - Ha f'_{m-1}(\eta),$ (46)
$S_m(\eta) = \left(\frac{3+4R}{Pr}\right) \theta''_{m-1}(\eta) + \sum_{k=0}^{m-1} f_{m-1-k}(\eta)\theta'_k(\eta) + N_b \sum_{k=0}^{m-1} \varphi'_{m-1-k}(\eta)\theta'_k(\eta) -$
$N_t \sum_{k=0}^{m-1} \theta'_{m-1-k}(\eta)\theta'_k(\eta),$ (47)
$T_m(\eta) = \varphi''_{m-1}(\eta) + Le \sum_{k=0}^{m-1} f_{m-1-k}(\eta)\, \xi'_k(\eta) + \frac{N_t}{N_b} \theta''_{m-1}(\eta),$ (48)

with boundary conditions
$f_m(0) = 0\ f'_m(0) = 0\ f'_m(\infty) = 0,$ (49)
$\theta_m(0) = 0, \theta_m(\infty) = 0,$ (50)
$\varphi_m(0) = 0, \varphi_m(\infty) = 0.$ (51)

We solve these non-linear equations given by (43), (44) and (45) for f_m, θ_m, φ_m by MATHEMATICA. For the given equations we get the solution for the different characteristic parameters we analyze the solutions through graphs.

IV. RESULT AND DISCUSSION

The solutions for the (46)-(48) system of equation with corresponding boundary conditions are obtained by homotopy analysis method. With the help of Mathematica equations are solved and discussed through graphs. The convergence of the equation depends on varying auxiliary linear operator L, auxiliary parameter h and initial solution. The velocity profile $\boldsymbol{\phi}'(\boldsymbol{\eta})$ of nanofluid decreases by increase in the value of Hartman number Ha as depicted in fig3. In fig 4 we observed that by increase in the value of velocity parameter λ the fluid velocity increases. Fig5 gives the effect of Hartman number Ha on the temperature profile $\boldsymbol{\psi}(\boldsymbol{\eta})$, which increases with the increase in the value of magnetic field parameter Ha, here the applied magnetic field tends to heat the fluid which reduces the heat transfer from the wall. Whereas The effect of magnetic field parameter Ha lowers the value of nanoparticle volume fraction as in fig 9. In fig 6 Temperature profile $\boldsymbol{\psi}(\boldsymbol{\eta})$ increases by increase in the value of radiation parameter R, the thermal radiation enhances the thermal diffusion. In fig 7 and fig 8 we notice, the influence of thermophorosis parameter Nt and of Brownian motion parameter Nb enhance the temperature $\boldsymbol{\psi}(\boldsymbol{\eta})$ of the nanofluid. Whereas increases in the value of Lewis number Le results in thinning of the boundary layer which can be shown in fig 10. In fig 11 increase in the value of thermophorosis parameter Nt increases the nanoparticle volume fraction profile.

V. CONCLUSION

Present study we have applied HAM where the solution exactly matching with numerical result obtained by previous [36] author in which spectral relaxation method is applied directly to ODE. Here we observed that the semi analytical method HAM works well for non-linear differential equations

VI. GRAPHS

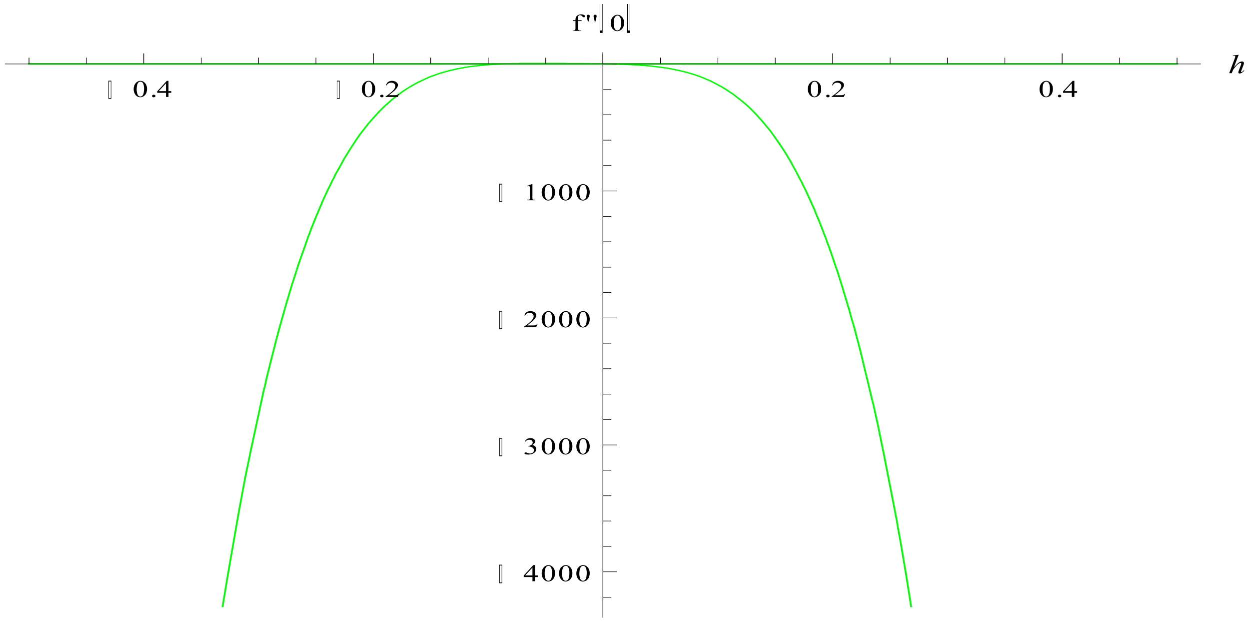

Figure 2 h-curve for velocity profile

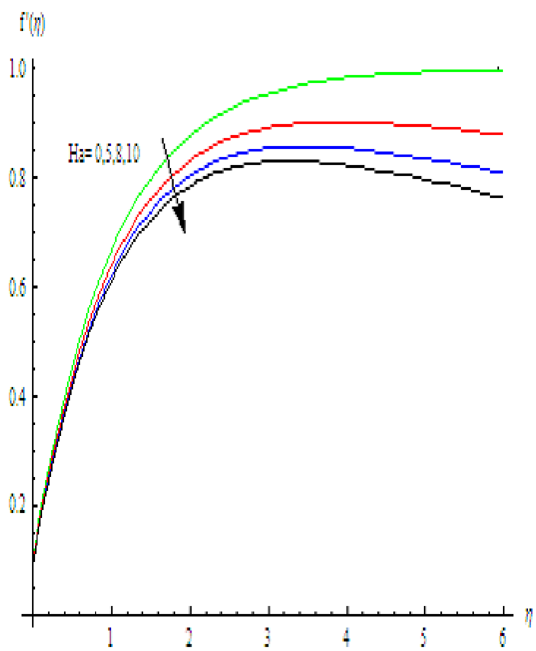

Fig3. Velocity profiles for different values of Ha

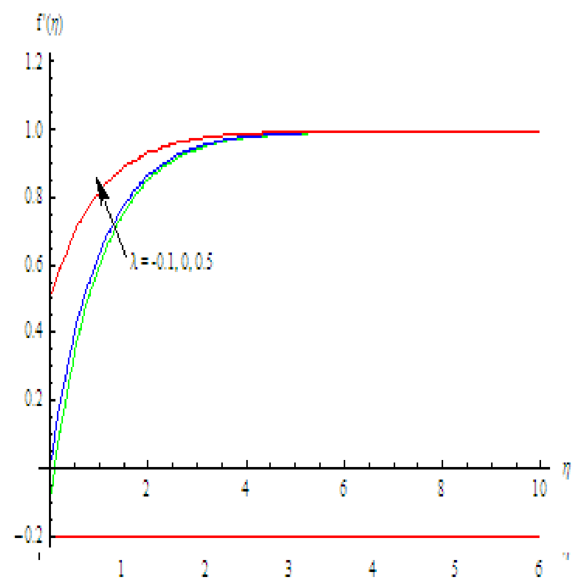

Fig4. Velocity profiles for different values of λ

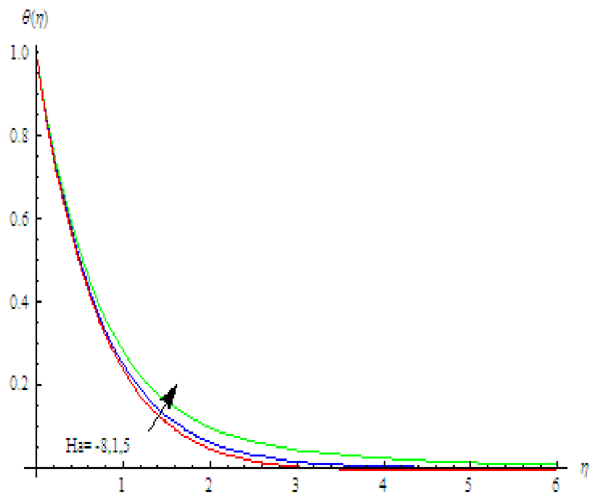

Fig5. Temperature distribution for different values of Ha

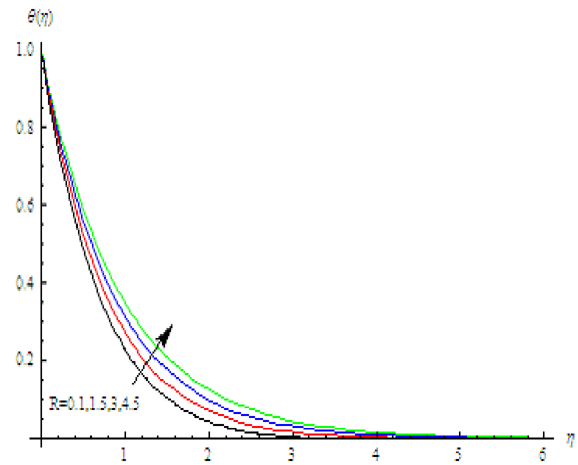

Fig6 Temperature distribution for different values of R

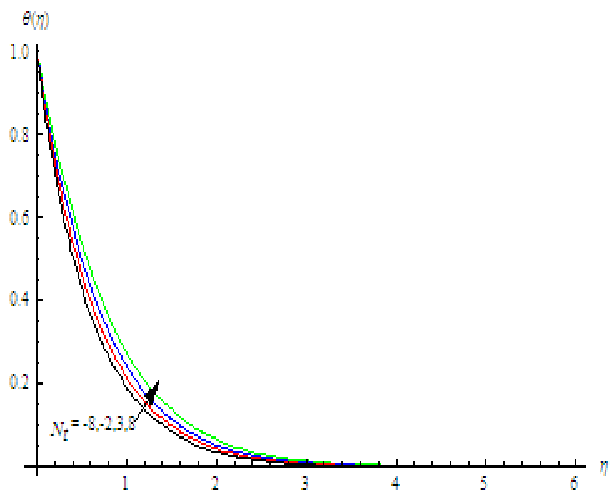

Fig7. Temperature distribution for different values of N_t

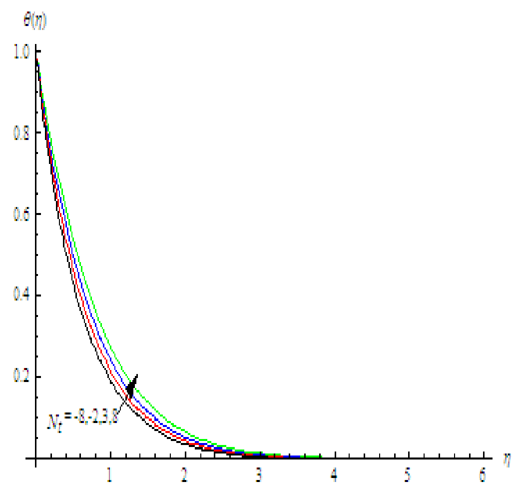

Fig8 Temperature distribution for different values of N_b

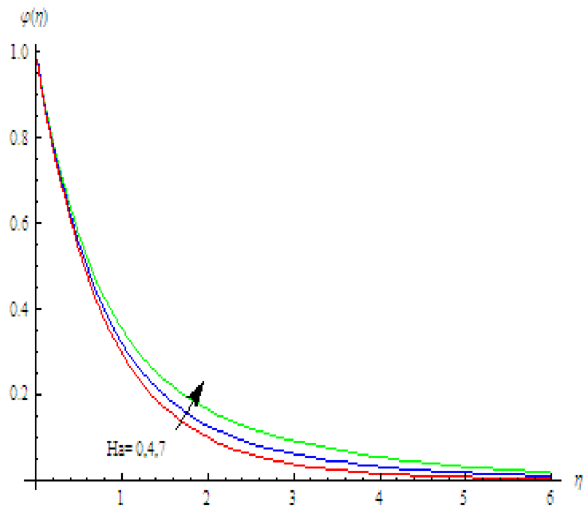

Fig9. Nano particle fraction for different values of Ha

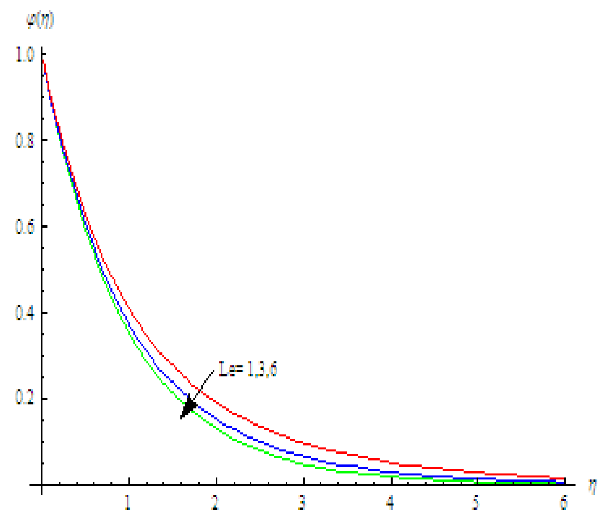

Fig10. . Nano particle fraction for different values of Le

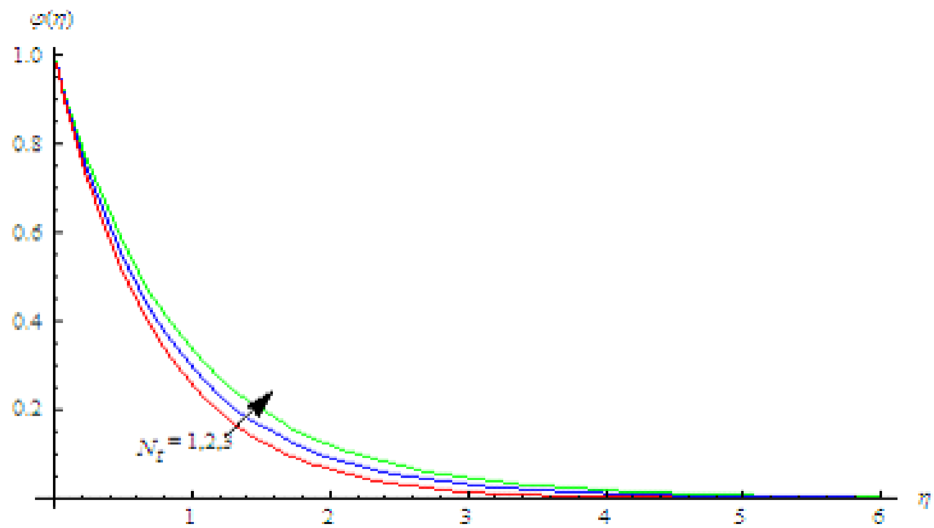

Fig11. Nano particle fraction for different values of N_t

Nomenclature:

u Velocity components along the x-axis

v Velocity components along the y-axis,

ν Kinematic viscosity coefficient,

k Thermal conductivity,

D_B Brownian diffusion coefficient,

D_T Thermophoresis diffusion coefficient,

B0 Uniform magnetic field strength of the base fluid,

σ Electrical conductivity of the base fluid,

α = $k/(\rho c)_f$ is the Thermal diffusivity of the fluid,

Pr Prandtl number

Nt Thermophoresis parameter

Ha Hartman number

τ_w Shear stress

R Radiation parameter

Le Lewis number

q_w Heat flux

q_m Mass flux

q_r Radiative heat flux

Nb Brownian motion parameter

τ Ratio of the nanoparticle heat capacity and the base fluid heat capacity.

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