CUBE DIFFERENCE LABELING OF SOME SPECIAL GRAPH FAMILIES

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ABSTRACT

A new labeling and a new graph called cube difference labeling and the cube difference is defined. Let G be a (p,q) graph. G is said to have a cube difference labeling if there exists injection **f:V(G)⟶{0,1,2,…,p-1}** such that the edge set of G has assigned a weight defined by the absolute cube difference of its end vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph. The cube difference labeling for some special graph families like **Pan graph**, **Lollipop graph, Barbell graph, Sunlet graph, Sparkler graph, Fan graph, Triangular Snake Graph, Z-P_n graph** are discussed in this paper.

Keywords: *Cube difference labeling, Cube difference graph.*

I. INTRODUCTION

All graph in this paper are simple finite undirected and nontrivial graph **G = (V,E)** with vertex set V and the edge set E. A function f is a cube difference labeling of a graph G of size n if f is an injection from V(G) to the set {0,1,2,….,p-1} such that, when each edge uv of G has assigned the weight $|[f(u)]^3-[f(v)]^3|$,the resulting weights are distinct. The notion of square difference labeling was introduced by J.Shima [4]-[6]. Graph labeling can also be applied in areas such as communication network, mobile telecommunications, and medical field. A dynamic survey on graph labeling is regularly updated by Gallian [2] and it is published by Electronic Journal of Combinatory. The notation and terminology used in this paper are taken from [1].

Definition 1.1: Let G = (V(G),E(G)) be a graph. G is said to be cube difference labeling if there exist a injection f:V(G)⟶{0,1,2,…,p-1} such that the induced function f*:E(G) ⟶N given by $f^*(uv) = |[f(u)]^3-[f(v)]^3|$ is injection.

Definition 1.2: A graph which satisfies the cube difference labeling is called the cube difference graph.

Definition 1.3: The Pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge. It is denoted by $\mathbf{P_n}$.

Definition 1.4: The Lollipop graph is the graph obtained by a Complete graph K_m to a path P_n with a bridge. It is denoted by $\mathbf{L_{m,n}}$.

Definition 1.5: The Barbell graph is obtained by connecting two copies of K_n by a bridge. It is denoted by $\mathbf{B_n}$.

Definition 1.6: The Sunlet graph $\mathbf{S_n}$ is a graph obtained from a cycle C_n attached a pendent edge at each vertex of the n-cycle. It has 2n vertices and 2n edges.

Definition 1.7: The Sparkler graph $\mathbf{P_m^{+n}}$ is a graph obtained from a path P_m and appending n edges to an end point. It has m+n vertices and m+n-1 edges.

Definition 1.8: A fan graph obtained by joining all the vertices of a path $\mathbf{P_n}$ to a further vertex, called the Centre. It is denoted by $\mathbf{F_n}$. It has n+1 vertices and 2n-1 edges.

Definition 1.9: The Triangular Snake $\mathbf{T_n}$ is obtained the path P_n by replace each of the path by a triangle. It has 2n+1 vertices and 3n edges.

Definition 1.10: In a pair path P_n, i^{th} vertex of a path P_1 is joined with $i+1^{th}$ vertex of a path P_2. It is denoted by **Z-P_n**.

II. MAIN RESULT

Theorem: 2.1
The Pan graph $\mathbf{P_n}$ admits a Cube difference labeling.

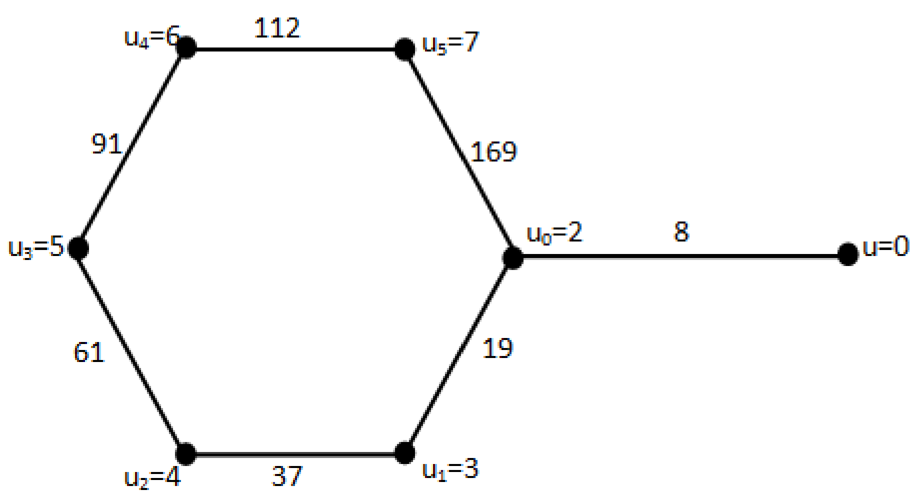
Proof:
Let $\mathbf{P_n}$ be a Pan graph. Let $|V(G)| = n+1$ and $|E(G)| = n+1$.
The mapping $\mathbf{f:V(G) \longrightarrow \{0,1,2,\dots,n-1\}}$ is defined by
$f(u) = 0$ and $f(u_i) = i+2$, $0 \le i \le n-1$ and the induced function, $f^*: E(G) \longrightarrow N$ is defined by
and here the edge sets are $E_1=\{u_iu_{i+1} \,/\, 0\le i\le n-1\}$ and $E_2=\{uu_i \,/\, i=1\}$
and the edge labeling are,

(i) $f^*(u_iu_{i+1}) = \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3|$
$= \bigcup_{i=0}^{n-1} |(i+1)^3 - (i+3)^3|$
$= \bigcup_{i=0}^{n-1} (3i^2+15i+19)$
$= \{19, 37, 61, \dots\}$

(ii) $f^*(uu_0) = (i+2)^3$, $i=0$
$= 8.$

Here all the edges are distinct. Hence, the Pan graph P_n admits a Cube difference labeling.

Example 2.2: The Pan graph $\mathbf{P_6}$ is a cube difference graph.



Theorem: 2.3
The Lollipop graph $\mathbf{L_{m,n}}$ admits a Cube difference labeling.

Proof:
Let $\mathbf{L_{m,n}}$ be a Lollipop graph. Let $|V(G)| = m+n$ and $|E(G)| = m+n+2$.
The mapping $\mathbf{f:V(G) \longrightarrow \{0,1,2,\dots,n-1\}}$ is defined by
$f(u_i) = i$, $0\le i\le n-1$ and $f(v_i)=i+1$, $n-1\le i\le 2(m-1)$ the induced function,
$f^*: E(G) \longrightarrow N$ is defined by and here the edge sets are $E_1=\{u_iu_{i+1} \,/\, 0\le i\le n-1\}$ and $E_2=\{v_iv_{i+1} \,/\, n\le i\le 2(m-1)\}$, $E_3=\{v_iv_{i+2} \,/ i=3\}$ and $E_4=\{v_{i+2}v_{i+4} \,/ i=2\}$ and the edge labeling are,

(i) $f^*(u_iu_{i+1}) = \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3|$
$= \bigcup_{i=0}^{n-1} |(i)^3 - (i+1)^3|$
$= \bigcup_{i=0}^{n-1} (3i^2+3i+1) =$
$= \{1, 7\}.$

(ii) $f^*(v_iv_{i+1}) = \bigcup_{i=1}^{m} |(f(v_i))^3 - (f(v_{i+1}))^3|$
$= \bigcup_{i=1}^{m} (3i^2+3i+7).$

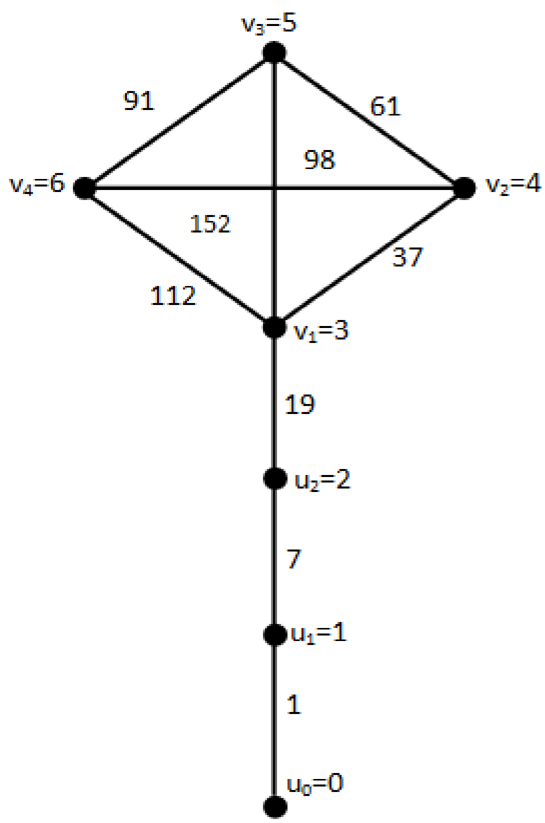
$=\{19,37,61,91,112\}$.

(iii) $f^*(v_iv_{i+2})=|(f(v_i))^3-(f(v_{i+2}))^3|$
$=|(i)^3-(i+2)^3|$
$=6i^2+24i+26.$ $,i=2$
$=98$

(iv) $f^*(v_{i+1}v_{i+3})= |(f(v_{i+1}))^3-(f(v_{i+3}))^3|$
$=|(i+2)^3-(i+4)^3|$
$=6i^2+36i+56.$
$=152.$

Here all the edges are distinct. Hence, the Lollipop graph $\mathbf{L_{m,n}}$ admits a Cube difference labeling.

Example 2.4: $\mathbf{L_{4,3}}$



Theorem: 2.5

The Barbell graph $\mathbf{B_n}$ admits a Cube difference labeling.

Proof:

Let $\mathbf{B_n}$ be the Barbell graph. Let $|V(G)|=2n$ and $|E(G)|=2n+1$.

The mapping $\mathbf{f:V(G) \longrightarrow \{0,1,2,\ldots,2n-1\}}$ is defined by $f(u_i)=i+1$, $0\leq i\leq 2n-1$. and induced function $f^*:E(G)\longrightarrow N$ is defined by, and here the sets are,

$E_1=\{u_iu_{i+1} /0\leq i\leq n-1\}$ and $E_2=\{u_iu_{i+2} /i=1\}$ and $E_3=\{u_{i+2}u_{i+4} /i=2\}$.

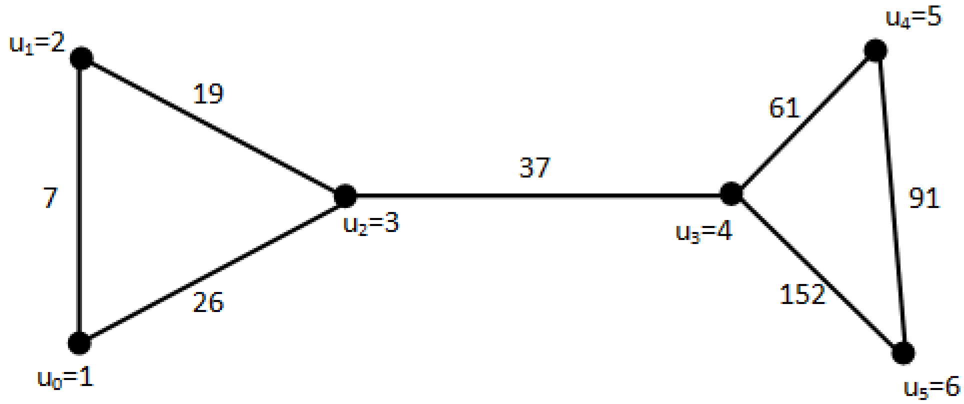
(i) $f^*(u_iu_{i+1}) =\cup_{i=0}^{n-1} |(f(u_i))^3-(f(u_{i+1}))^3|$
$=\cup_{i=0}^{n-1} |(i+1)^3-(i+2)^3|$
$=\cup_{i=0}^{n-1}(3i^2+9i+7)$
$=\{1,7,19,37,\ldots,91\}$

(ii) $f^*(u_iu_{i+2})=|i^3-(i+2)^3|$
$=6i^2+12i+8$ $,i=1$
$=26$

(iii) $f^*(u_{i+2}u_{i+4})=|(f(u_{i+2}))^3-(f(u_{i+4}))^3|$
$=|(i+2)^3-(i+2)^3|$
$=6i^2+36i+56 \quad ,i=2$
$=152.$

Hence all the edges are distinct. Hence the graph $\mathbf{B_n}$ admits a Cube difference labeling.

Example2.6: The Barbell graph $\mathbf{B_3}$ is a Cube difference graph



Theorem: 2.7
The Sunlet graph $\mathbf{S_n}$ admits a Cube difference labeling.

Proof:
Let $\mathbf{S_n}$ be a Sunlet graph. Let $|V(G)|=2n$ and $|E(G)|=2n$.
The mapping $\mathbf{f:V(G) \longrightarrow \{0,1,2,\ldots,2n-1\}}$ is defined by $f(u_i)=i \quad , 0\leq i\leq 2n-1$
and the induced function $f^*:E(G)\longrightarrow N$ is defined by, and here the sets are,
$E_1=\{u_iu_{i+1} / 0\leq i\leq n-1\}$ and $E_2=\{u_{n-1}u_0\}$
$E_3=\{u_iu_{n+I} / 0\leq n+i\leq 2n-1\}$ and the edge labeling are,

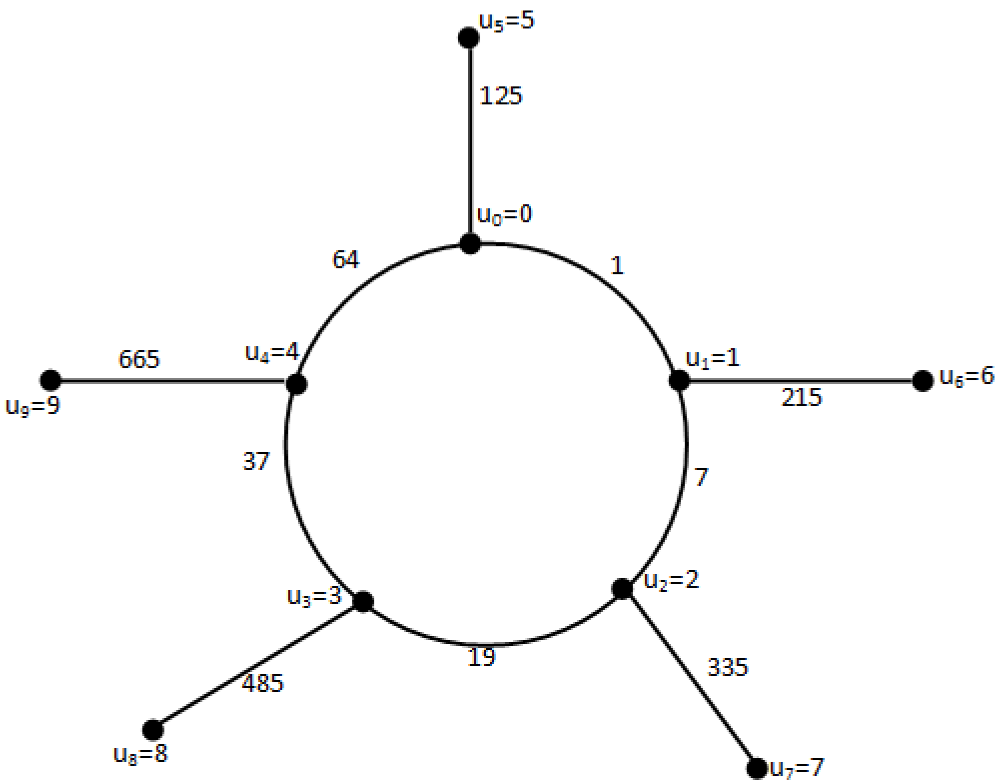
(i) $f^*(u_iu_{i+1}) = \bigcup_{i=0}^{n-1} |(f(u_i))^3-(f(u_{i+1}))^3|$
$= \bigcup_{i=0}^{n-1}(3i^2+3i+1)$
$=\{1,7,19,37\}$

(ii) $f^*(u_{n-1}u_0)=(n-1)^3$
$=64.$

(iii) $f^*(u_iu_{n+i})= \bigcup_{i=0}^{n-1} |(f(u_i))^3-(f(u_{n+i}))^3|$
$= \bigcup_{i=0}^{n-1}(15i^2+75i+125)$
$=\{125,215,335,485,665\}$

Here all the edges are distinct. Hence the Sunlet graph $\mathbf{S_n}$ admits a Cube difference labeling.

Example 2.8: The Sunlet graph $\mathbf{S_5}$ is a Cube difference graph.



Theorem: 2.9
A Sparkler graph $\mathbf{P_m^{+n}}$ admits a Cube difference labeling.

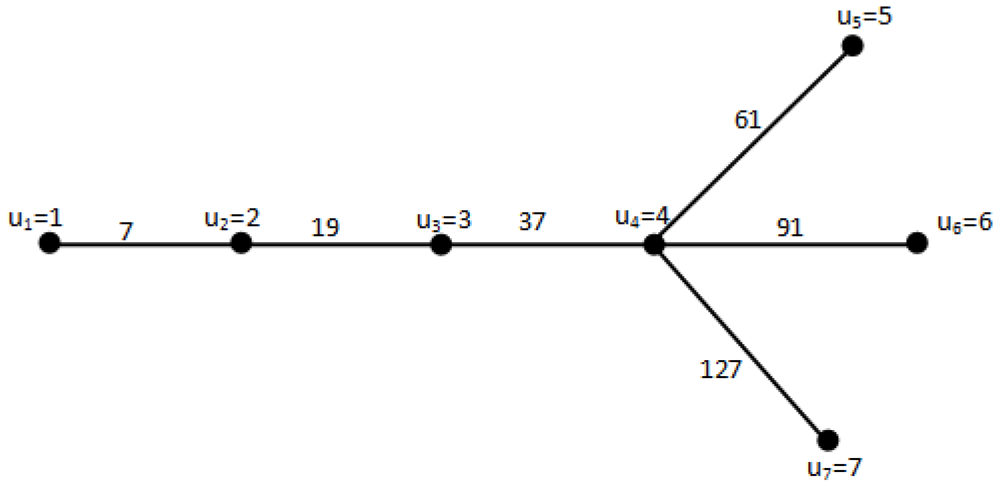
Proof:
Let $\mathbf{P_m^{+n}}$ be a Sparkler graph. Let $|v(G)|=m+n$ and $|E(G)|=m+n-1$.
The mapping $\mathbf{f:V(G)\longrightarrow\{0,1,2,\ldots,n-1\}}$ is defined by $f(u_i)=i$ $,1\leq i\leq m$
and $f(u_j)=m+1$ $,m+1\leq j\leq 2n+1$, and the induced function, $f^*:E(G)\longrightarrow N$ is defined by, and here the sets are, $E_1=\{u_iu_{i+1} / 1\leq i\leq m-1\}$, $E_2=\{u_iv_j / i=m ,m+1\leq j\leq 2n+1\}$ and the edge labeling are

(i) $f^*(u_iu_{i+1}) = \bigcup_{i=1}^{m} |(f(u_i))^3-(f(u_{i+1}))^3|$
$= \bigcup_{i=1}^{m}(3i^2+3i+1)$
$=\{7,19,37\}$

(ii) $f^*(u_iu_j) = |(f(u_i))^3-(f(v_j))^3|$ $,i=m$ and $m+1\leq j\leq n$
$= \bigcup_{i=m+1}^{2n+1}(3i^2+3i+1)$
$=\{61,91,127\}$

Here all the edges are distinct. Hence the Sparkler graph $\mathbf{P_m^{+n}}$ admits a Cube difference labeling.

Example 2.10: The Sparkler graph $\mathbf{P_4^{+3}}$ is a Cube difference graph.



Theorem: 2.11
The Fan graph $\mathbf{F_n}$ admits a Cube difference labeling.

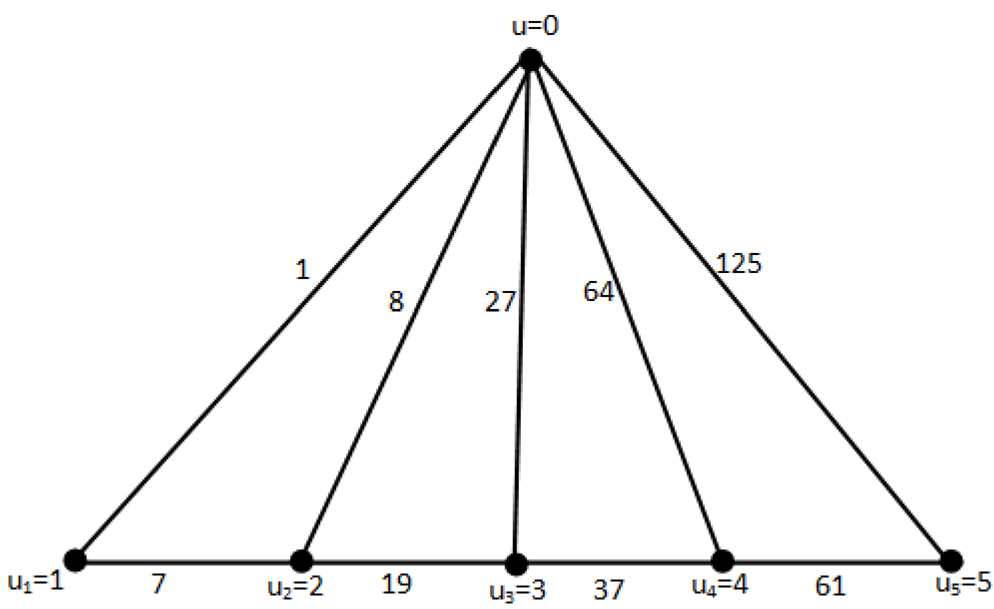
Proof:
Let F_n be a Fan graph. Let $|V(G)|=n+1$ and $|E(G)|=2n-1$.
The mapping $\mathbf{f:V(G) \longrightarrow \{0,1,2,\ldots,n-1\}}$ is defined by $f(u)=0$ and $f(u_i)=i$, $1\leq i\leq n$
and the induced function $f^*:E(G)\longrightarrow N$ is defined by, and here the sets are,
$E_1=\{u_iu_{i+1} / 1\leq i\leq n-1\}$ and $E_2=\{uu_i / 1\leq i\leq n\}$ and the edge labelings are,

(i) $f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1} |(f(u_i))^3-(f(u_{i+1}))^3|$
$= \bigcup_{i=1}^{n-1}(3i^2+3i+1)$
$=\{7,19,37,61\}$

(ii) $f^*(uu_i) = \bigcup_{i=1}^{n} |(f(u))^3-(f(u_i))^3|$
$= \bigcup_{i=1}^{n}(i)^3$
$=\{1,8,27,64,125\}$

Here all the edges are distinct. Hence the Fan graph $\mathbf{F_n}$ admits a Cube difference labeling.

Example 2.12: The Fan graph $\mathbf{F_5}$ is a Cube difference graph.



Theorem: 2.13

A Triangular Snake graph $\mathbf{T_n}$ admits a Cube difference labeling.

Proof:
Let $\mathbf{T_n}$ be a Triangular Snake graph. Let |V(G)|=2n+1 and |E(G)|=3n.
The mapping **f:V(G) ⟶{0,1,2,….,2n-1}** is defined by $f(u_i)=2i$, $0\le i\le n-1$
and $f(v_i)=2i+1$, $0\le i\le n-1$ and the induced function, f*:E(G)⟶N is defined by,
and here the sets are, $E_1=\{v_iv_{i+1}/0\le i\le n-1\}$, $E_2=\{u_iv_i/0\le i\le n-1\}$ and
$E_3=\{u_iv_{i+1}/0\le i\le n-1\}$ and the edge labelings are,

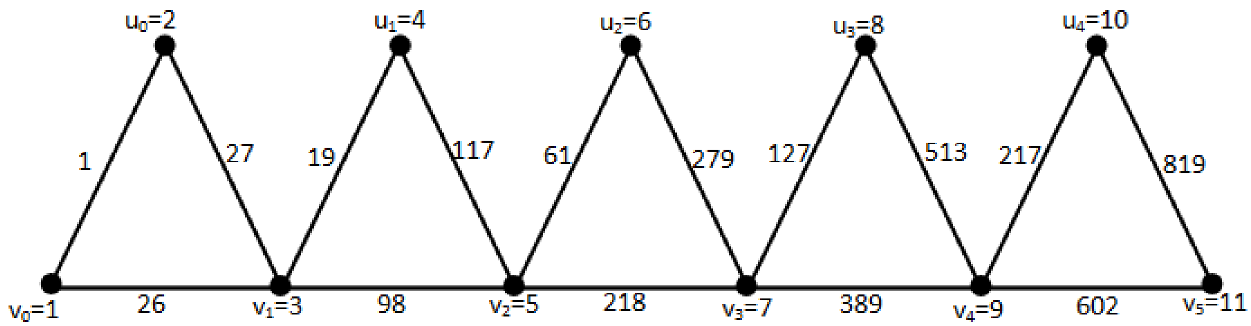
(i) $f^*(v_iv_{i+1}) =\bigcup_{i=0}^{n-1} |(f(v_i))^3-(f(v_{i+1}))^3|$
$=\bigcup_{i=0}^{n-1} |(2(i+1))^3-(2(i+1)+1))^3|$
$=\bigcup_{i=0}^{n-1}(24i^2+48i+26)$
$=\{26,98,218,386,602\}$.

(ii) $f^*(u_iv_i) =\bigcup_{i=0}^{n-1} |(f(u_i))^3-(f(v_i))^3|$
$=\bigcup_{i=0}^{n-1} |(2i)^3-(2i+1)^3|$
$= \bigcup_{i=0}^{n-1}(12i^2+6i+1)$
$=\{1,19,61,127,217\}$

(iii) $f^*(u_iv_{i+1})=\bigcup_{i=0}^{n-1} |(f(u_i))^3-(f(v_{i+1}))^3|$
$= \bigcup_{i=0}^{n-1}(36i^2+54i+27)$
$=\{27,117,279,513,819\}$

Here all the edges are distinct. Hence the Triangular Snake graph $\mathbf{T_n}$ admits a Cube difference labeling.

Example 2.14: The Triangular Snake graph $\mathbf{T_5}$ is a Cube difference graph.



Theorem: 2.15
The **Z-P$_n$** graph admits a Cube difference labeling.

Proof:
Let Z-P$_n$ be a graph. Let |V(G)|=2n. The mapping **f:V(G) ⟶{0,1,2,….,2n-1}** is defined by
$f(u_i)=2i$, $0\le i\le n-1$ and $f(v_i)=2i+1$, $0\le i\le n-1$ and the induced function f*:E(G)⟶N is defined by, and here the sets are,
$E_1=\{u_iu_{i+1}/0\le i\le n-1\}$, $E_2=\{v_iv_{i+1}/0\le i\le n-1\}$ and $E_3=\{v_iu_{i+1}/0\le i\le n-1\}$ and the edges labelings are

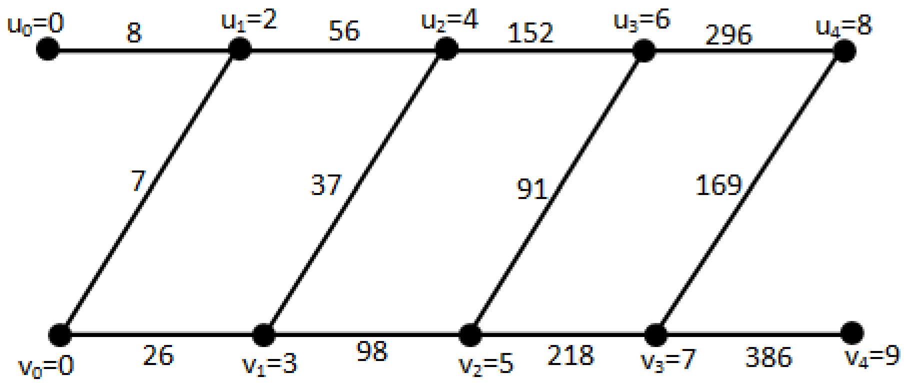
(i) $f^*(u_iu_{i+1}) =\bigcup_{i=0}^{n-1} |(f(u_i))^3-(f(u_{i+1}))^3|$
$=\bigcup_{i=0}^{n-1}(24i^2+24i+8)$
$=\{8,56,152,296\}$

(ii) $f^*(v_iv_{i+1}) =\bigcup_{i=0}^{n-1} |(f(v_i))^3-(f(v_{i+1}))^3|$
$= \bigcup_{i=0}^{n-1}(24i^2+48i+26)$
$=\{26,98,218,386\}$

(iii) $f^*(v_iu_{i+1})=\bigcup_{i=0}^{n-1} |(f(v_i))^3-(f(u_{i+1}))^3|$

$=\cup_{i=0}^{n-1}(12i^2+18i+7)$
$=\{7,37,91,169\}$
Here all the edges are distinct. Hence **Z-P$_n$** admits a Cube difference labeling.

Example 2.16: The **Z-P$_5$** graph is a Cube difference graph.



III. CONCLUSION

In this paper the Special graphs, are investigated for the Cube difference labeling. This labeling can be verified for some other graphs.

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