STRONGLY CHROMATIC METRO DOMINATION OF P_n, C_n AND ${P_n}^2$

María del Pilar Gómez, Diego A. Muñoz
Department of Physics, Universidad de Chile, Santiago, Chile

ABSTRACT

A dominating set *D* of a graph *G(V,E)* is called metro dominating set *G* if for every pair of vertices *u,v*, there exists a vertex *w* in *D* such that *d(u,w) ≠ d(v,w)*. A metro dominating set *D* is called strongly chromatic metro dominating set if for every vertex *v*∈ *D* is from the same color class. The minimum cardinality strongly chromatic metro dominating set is called strongly chromatic metro domination number and is denoted by $SC\gamma_\beta$. In this paper we find strongly chromatic metro domination number of path, cycles and square of a path.

Keywords: *metric dimension, metro domination, strongly chromatic metro domination, power graph.*
AMS Mathematics Subject Classification (2010):05C56.

I. INTRODUCTION

Let *G(V,E)* be a simple, non-trivial, undirected and non-null graphs. A graph *G* is k-colorable if there exists a k-coloring of *G*. One of the fastest growing areas within graph theory is the study of domination and related problem. A subset *D* of *V* is said to be a dominating set of *G* if every vertex in *V-D* is adjacent to a vertex in *D*.
The minimum cardinality of a dominating set is called the domination number of *G* and is denoted by $\gamma(G)$. A subset *D*of *V* is said to be a dom-chromatic set if*D* is a dominating set and $\chi(<D>) = \chi(G)$.The dom-chromatic number $\gamma_{ch}(G)$ of *G* is the minimum cardinality of a dom-chromatic set.

In 1976 F.Harary and R.A.Melter [1] introduced the notation of metric dimension. A vertex *x*∈ *V(G)* resolves a pair of vertices *u,w* ∈ *V(G)* if *d(v,x) ≠ d(w,x)*. A set of vertices *S* ⊆*V(G)* resolves *G* and *S* is a resolving set of *G*, if every pair of distinct vertices of *G* are resolved by same vertex in *S*. A resolving set *S* of *G* with minimum cardinality is a metric dimension of *G* denoted by $\beta(G)$.

A dominating set *D* of *V(G)* having a property that for each pair of vertices *u,v* there exist a vertex *w* in *D* such that *d(u,w) ≠ d(v,w)* is called metro dominating set of *G* or simply MD-set. The minimum cardinality of a metro dominating set of *G* is called metro domination number of *G* and is denoted by $\gamma_\beta(G)$.

II. DEFINITIONS

2.1 Metric dimension:
The metric dimension of a graph *G* is the minimum cardinality of a subset *S* of vertices such that all other vertices are uniquely determined by their distances to the vertices in *S*. It is denoted by $\beta(G)$.

2.2 Domination:
Let *G(V,E)* be a graph. A subset of vertices *D* ⊆ *V* is called a dominating set (γ-set) if every vertex in *V-D* adjacent to atleast one vertex in *D*. The minimum cardinality of a dominating set is called the domination number of the graph *G* and is denoted by $\gamma(G)$.

2.3 Locating domination:
A dominating set *D* is called a locating dominating set or simply LD-set if for each pair of vertices *u,v* ∈ *V-D, ND(u)* ≠ *ND(v)* where *ND(u)* = *N(u)*∩*D*. The minimum cardinality of an LD-set of the graph *G* is called the locating domination number of *G* denoted by $\gamma_L(G)$.

2.4 Metro domination:
A dominating set *D* of *V(G)* having the property that for each pair of vertices *u,v* there exists a vertex *w* in *D* such that *d(u,w) ≠ d(v,w)* is called metro dominating set of *G* or simply MD-set. The minimum cardinality of a metro dominating set of *G* is called metro domination number of *G* and is denoted by $\gamma_\beta(G)$.

2.5 Chromatic number:
The minimum number of colors required for a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$.

2.6 Chromatic domination:
A subset D of V is said to be a dom-chromatic set if D is a dominating set and $\chi(<D>) = \chi(G)$. The dom-chromatic number $\gamma_{ch}(G)$ of G is the minimum cardinality of a dom-chromatic set.

III. SOME KNOWN RESULTS

In this section we mention some of the known result on metric dimension, domination, metro domination.

Theorem 3.1. (Harary and Melter [1]) The metric dimension of a non trivial complete graph of order $n$is n-1.

Theorem 3.2. (Khuller, Raghavachari, Rosenfeld [4]) The metric dimension of a graph G is 1 if and only if G is a path.

Theorem 3.3. (Harary and Melter [1]) The metric dimension of a complete bipartite graph $K_{m,n}$is $m+n$-2.

Theorem 3.4.[5] The metro domination number of a graph G is $\left\lceil\frac{n}{5}\right\rceil$ if and only if G is a cycle.

Theorem 3.5.[5] Let G be a graph on n vertices. Then $\gamma_\beta(G) = n$-1 if and only if G is K_n or $K_{1,n\text{-}1}$ for$n \geq 1$.

Theorem 3.6. [5] For any integer n, $\gamma_\beta(P_n) = \left\lceil\frac{n}{3}\right\rceil$.

Remark 3.7. For any connected graph G, $\gamma_\beta(G) \geq \max\{\gamma(G), \beta(G)\}$.

Remark 3.8. For any integer $n>3$, $\chi(C_n) = \begin{cases} 3 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$

Remark 3.9. For any integer $n> 1$, $\chi(P_n) = 2$.

Lemma 3.10. [9] Let $G = P_n^2$, $n> 3$. Then $dim(G) = 2$.

Theorem 3.11.[7]For every $n \geq 1$, $\gamma_\beta(P_n^2) = \left\lceil\frac{n}{5}\right\rceil$.

Theorem 3.12. [2] For any integer $n \geq 3$,$\gamma_\beta(P_n^2) = \begin{cases} 2 & \text{if } 3 \geq n \geq 7 \\ 3 & \text{if } 8 \geq n \geq 10 \\ \left\lceil\frac{n}{5}\right\rceil & \text{if } n \geq 11 \end{cases}$

Remark 3.13. For any integer $n \geq 3$,$\chi(P_n^2) = 3$.

IV. MAIN RESULTS

Theorem 4.1. For any integer$n \geq 4$, $SC\gamma_\beta(P_n) = \left\lceil\frac{n-1}{2}\right\rceil$.

Proof: By theorem 3.2 $\beta(P_n) = 1$ and by remark 3.9 $\chi(P_n) = 2$, clearly we have $\left\lceil\frac{n}{2}\right\rceil$ vertices of one color class and remaining $\left\lceil\frac{n}{2}\right\rceil$ vertices of other color class. Hence we have choice of either $\left\lceil\frac{n}{2}\right\rceil$ or $\left\lceil\frac{n}{2}\right\rceil$ vertices for dominating set D

whose vertices are from the same color class.For even n, $\frac{n}{2}$ vertices of same color class dominates the remaining $\frac{n}{2}$vertices. For odd$n$, $\frac{n-1}{2}$ vertices of same color class will dominates the remaining vertices and hence $SC\gamma_\beta(P_n) \geq \left\lceil\frac{n-1}{2}\right\rceil$ (1)

To prove the reverse inequality, we define a strongly chromatic dominating set$D = \left\{v_{2i} \,/\, 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$ of cardinality $\left\lceil\frac{n-1}{2}\right\rceil$. We note that Dacts as a dominating set also as a resolving set and each $v_i \in D$ is from the same color class and hence $SC\gamma_\beta(P_n) \leq \left\lceil\frac{n-1}{2}\right\rceil$ (2)

from(1) and (2)
$SC\gamma_\beta(P_n) = \left\lceil\frac{n-1}{2}\right\rceil$.

Theorem 4.2. For any integer $n \geq 5$, $SC\gamma_\beta(C_n) = \left\lceil\frac{n-1}{2}\right\rceil$.

Proof: By the result $\beta(C_n) = 2$ and by remark 3.8, $\chi(C_n) = \begin{cases} 3 & \text{for } n \text{ odd} \\ 2 & \text{for } n \text{ even} \end{cases}$, clearly we have $\frac{n}{2}$ vertices of one color class and remaining $\frac{n}{2}$ vertices of other color class for even $n$and $\left\lfloor\frac{n}{2}\right\rfloor$ vertices of one color class and other $\left\lfloor\frac{n}{2}\right\rfloor$ vertices of second color class and remaining one vertex of third color class for odd n. Hence we have choice of $\frac{n}{2}$ vertices for dominating set D such that each $v_i \in D$ are from the same color class.For even cycle, $\frac{n}{2}$vertices of same color class dominate the remaining $\frac{n}{2}$ vertices. For odd cycle, $\frac{n-1}{2}$ vertices of same color class will dominate the remaining vertices and hence $SC\gamma_\beta(C_n) \geq \left\lceil\frac{n-1}{2}\right\rceil$ (1)

To prove the reverse inequality, we define a strongly chromatic dominating set $D = \left\{v_{2i-1} \,/\, 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$ of cardinality $\left\lceil\frac{n-1}{2}\right\rceil$, which also acts as a resolving set and each$v_i \in D$ is from the same color class and hence $SC\gamma_\beta(C_n) \leq \left\lceil\frac{n-1}{2}\right\rceil$ (2)

from(1) and (2)
$SC\gamma_\beta(C_n) = \left\lceil\frac{n-1}{2}\right\rceil$.

Theorem 4.3. For any integer $n \leq 9$, $SC\gamma_\beta(P_n^2) = \left\lfloor\frac{n}{3}\right\rfloor$.

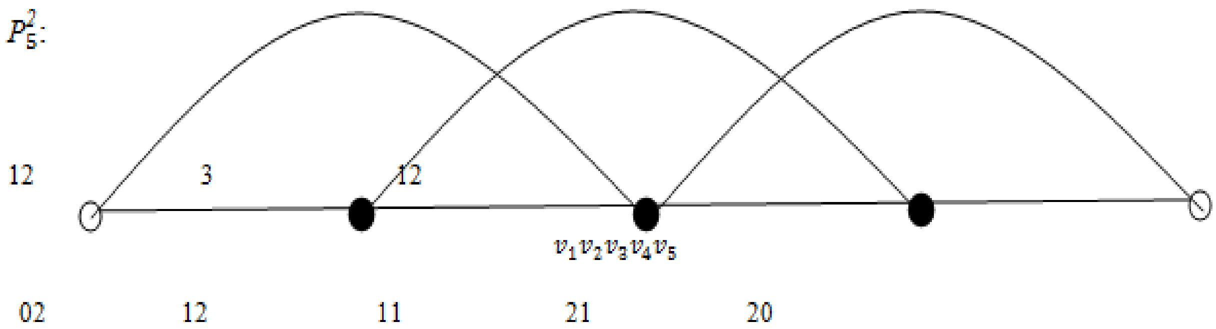
Proof: By lemma 3.10,$dim(P_n^2) = 2$ for $n > 3$. Also by Theorem 3.11 $\gamma_\beta(P_n^2) = \left\lceil\frac{n}{5}\right\rceil$, $n \geq 11$ here each $\left\lceil\frac{n}{5}\right\rceil$ vertices of metro dominating set are not from the same color class. By remark 3.13, $\chi(P_n^2) = 3$, $n \geq 3$ if we label v_1 of $P_n^2$by color 1 and v_2 by color 2 and v_3 by color 3 and continuing the coloring, we get $\left\lceil\frac{n}{3}\right\rceil$ vertices of color class 1, $\left\lfloor\frac{n+1}{3}\right\rfloor$ vertices of color class 2 and $\left\lfloor\frac{n}{3}\right\rfloor$ vertices of color class 3. Hence we have a choice of $\left\lceil\frac{n}{3}\right\rceil$ or $\left\lfloor\frac{n+1}{3}\right\rfloor$ or $\left\lfloor\frac{n}{3}\right\rfloor$ vertices for strongly chromatic metro dominating set minimum among these $\left\lfloor\frac{n}{3}\right\rfloor$is minimum and hence
$SC\gamma_\beta(P_n^2) \geq \left\lfloor\frac{n}{3}\right\rfloor$ (1)

To prove the reverse inequality, we defined the strongly chromatic metro dominating set as$D = \left\{v_{3i} \,/\, 1 \leq i \leq \left\lfloor\frac{n}{3}\right\rfloor\right\}$ of cardinality$\left\lfloor\frac{n}{3}\right\rfloor$. We note that D is a dominating set also acts as a resolving set and each $v_i \in D$ are all from the same color class and hence $SC\gamma_\beta(P_n^2) \leq \left\lfloor\frac{n}{3}\right\rfloor$(2)

from(1) and (2)

$SC\gamma_\beta(P_n^2) = \left\lfloor \frac{n}{3} \right\rfloor$.

EXAMPLE:



$D_1 = \{v_3\}$
D_1 is a dominating set but not resolving set.
$D_2 = \{v_1, v_5\}$
D_2 is a dominating set also resolving set but both vertices are not from same color class. Hence it is not a strongly chromatic metro domination.

Hence P_5^2 is not a strongly chromatic metro domination.

REFERENCE

1. *F.Harary and R.A. Melter, on the metric dimension of graphs, Ars Combinatoria, 2(1976) 191-195.*
2. *G.C. Basavaraju, M. Vishukumar and P. Raghunath, Metro domination of Square Path, Annals of Pure and Applied Mathematics vol.14, No. 3,2017,539-545.*
3. *S. Balamurugan, P.Aristotle, V. Swaminathan and G. Prabakaran, on graphs whose neighbourhood chromatic domination number is two, International Journal of Pure and Applied Mathematics, Volume 119, No. 15, 2018, 213-224.*
4. *S. Khuller, B. Raghavachari and A. Rosenfeld, Landmarks in graph, Discrete Appl. Math., 70 (3) (1996) 217-229.*
5. *P.R aghunath and B. Sooryanarana, Metro domination number of a graph, Twentieth annual conference of Rmanujan Mathematics Society, July 25-30 (2005) University of Calicut.*
6. *S. Lakshminarayana and M. Vishukumar, on the k-Metro domination number of Paths, Annals of Pure and Applied Mathematics, Vol. 14, No. 3, 2017, 593-600.*
7. *M. Alishahi and S.H. Shalmaee, Domination number of Square of Cartesian product of Cycles, Open Journal of Discrete Mathematics, 5(2015) 88-94.*
8. *G.C. Basavaraju, M. Vishukumar and P. Raghunath, Metro domination of Square Cycle, International Journal of Mathematics and its Applications, Volume 5, Issue 4-E(2017), 641-645.*
9. *M. M.AlHoli, O.A. Abughneim and H.Al. Ezeh, Metric dimension of Path related graphs, Global Journal of Pure and Applied Mathematics, 13 (2) (2017) 149-157.*